

**III B.TECH – I SEM EXAMINATIONS, NOVEMBER - 2010**  
**FINITE ELEMENT METHODS**  
**(MECHANICAL ENGINEERING)**  
**(MECHATRONICS)**

Time: 3hours

Max.Marks:80

Answer any FIVE questions  
 All questions carry equal marks

- - -

- 1.a) Explain the basic concept of FEM and list some of its advantages and applications.
- b) Derive the equilibrium equation in Cartesian coordinates. [6+10]
  
- 2.a) Derive the finite element equation using the potential energy approach.
- b) Derive the stiffness matrix for a bar element with one degrees of freedom at each node from basics. [8+8]
  
- 3.a) Derive the strain displacement matrix for a 2D beam element.
- b) Determine the deflection and slope under the point load for the beam shown in fig. 1.  
 $E = 210\text{GPa}; I = 4 \times 10^{-6} \text{ m}^4$ . [8+8]

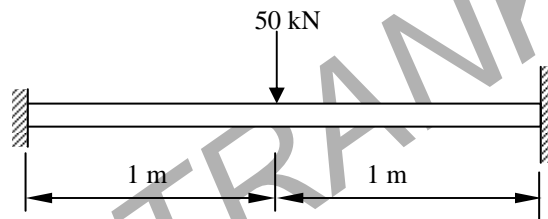


Fig. 1

- 4.a) Formulate the material property matrix  $[D]$  for a triangular element with  $E = 80 \text{ GPa}$  and  $\nu = 0.20$  using plane stress and plane strain conditions.
- b) Formulate the finite element equations for constant strain triangle element shown in fig.2. using plane stress assumption. [6+10]  
 $E = 200 \text{ GPa}; \nu = 0.20$ ; Thickness = 4 mm.

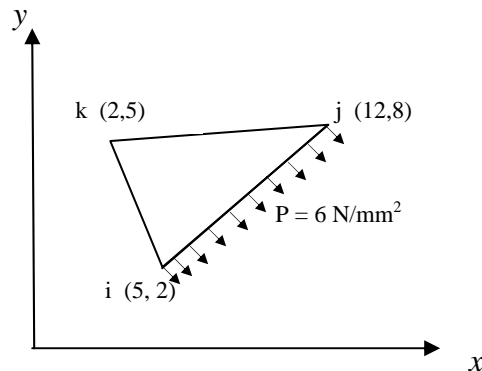


Fig. 2

5. Derive the strain displacement matrix for an axisymmetric problem. [16]
  
- 6.a) Explain in detail the applications of isoparametric elements in two and three dimensional stress analysis.
- b) Using Gaussian quadrature evaluate the following integral  $\int_{-1}^{+1} (4\xi + \xi^3) d\xi$ . [8+8]

7. Calculate the conductance matrix  $[K^{(e)}]$  and load vector  $\{F^{(e)}\}$  for the triangle element shown in fig.4 . The thermal conductivities are  $k_x = k_y = 6 \text{ W/cm}^\circ\text{C}$  and  $h = 0.4 \text{ W/cm}^2 \text{ }^\circ\text{C}$ . Thickness of the element is  $0.5 \text{ cm}$ . All coordinates are given in cms. Convection occurs on the side joining nodes  $i$  and  $j$ . [16]

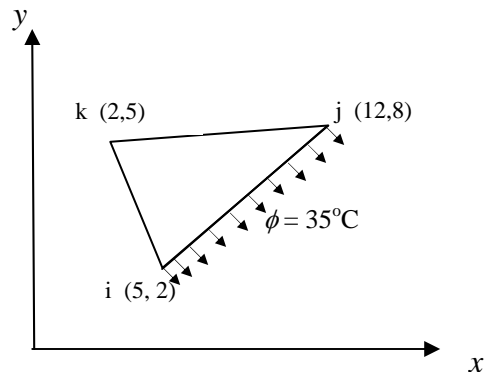


Fig. 4

8. Obtain the eigen values and eigen vectors for the cantilever beam of length 1m using constant mass for translation dof with  $E = 210\text{GPa}$ ,  $\rho = 7800\text{kg/m}^3$ . [16]

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- 1.a) What is the necessity of compatibility equations? Write the compatibility equations in Cartesian coordinates.
- b) Consider the following state of stress and strain:  
 $\sigma_{xx} = x^2$ ,  $\sigma_{yy} = y^2$ ,  $\epsilon_{xy} = -3xy$ ,  $\sigma_{zz} = \epsilon_{xz} = \epsilon_{yz} = 0$
- Determine whether the equilibrium equations are satisfied. [6+10]

- 2.a) Explain the different approaches for the treatment of boundary conditions in Finite Element Analysis.
- b) Determine nodal displacements, element stresses in the bar shown in fig.1. [6+10]

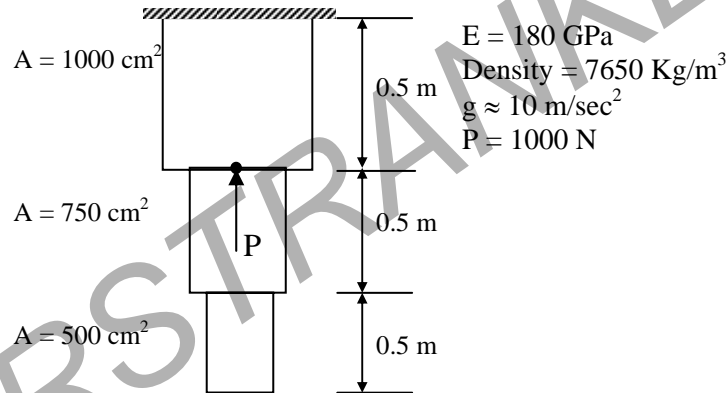


Fig. 1

- 3.a) Formulate global stiffness matrix and global load vector for the beam shown in fig.2
- b) State the boundary conditions. [16]

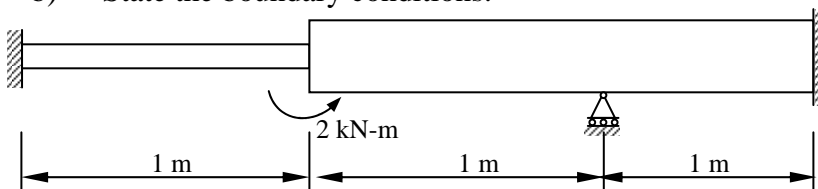


Fig. 2

- 4.a) Explain about constant strain triangle element. What are the limitations in using CST element.
- b) For the triangular element shown in fig.3, obtain the shape functions at point P (2, 2) within the element.

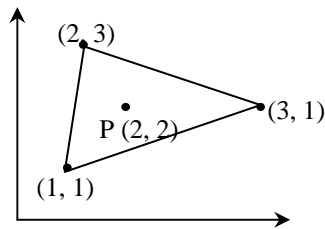


Fig. 3

- c) Derive the Jacobian matrix for a 3 noded triangular element. [4+6+6]
- 5.a) Differentiate between strain matrix for a plane strain problem and axisymmetric problem.
- b) The calculated displacements for an axisymmetric element with  $E = 110\text{GPa}$  and  $\nu = 0.20$  are given below:  
 $u_i = 0.01$ ;  $u_j = 0.04$ ;  $u_k = 0.02$ ;  $w_i = -0.02$ ;  $w_j = 0$  and  $w_k = 0.02$   
 The nodal coordinates in mm :  
 $r_i = 5$ ;  $r_j = 1$ ;  $r_k = 3$ ;  $z_i = 5$ ;  $z_j = 5$  and  $z_k = 2$   
 Calculate the resultant element stresses at the centroid of the element. [6+10]
6. The coordinates of each node of a four-node quadrilateral element are 1(3, 1); 2(4, 1); 3(5, 5) and 4(2, 4). The displacement vector of the element is given as  
 $\{q\} = \{0.20 \ 0 \ 0.10 \ 0 \ 0.12 \ 0.15 \ 0 \ 0.06\}^T$   
 Find  
 i) The x, y coordinates of a point whose location in the master element is given by  $\xi = 0.2$  and  $\eta = 0.4$   
 ii) The  $u, v$  displacement of point P  
 iii) The Jacobian  $[J]$  at  $\xi = \eta = 0.4$ . [16]
7. Find the temperature distribution in the one dimensional fin shown in fig. 4. [16]

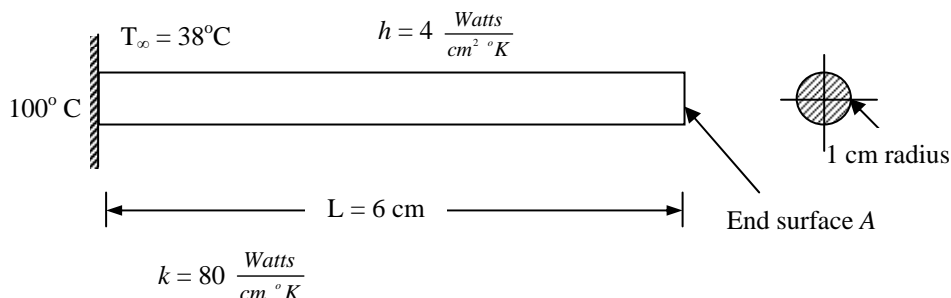


Fig. 4

8. Find the approximate first two natural frequencies of a simply supported beam using one element. [16]

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- 1.a) If displacement field is described by  
 $u = (3x^2 + y^2 + 2xy) 10^{-4}$ ,  $v = (4x - 6y^2 + 4y) 10^{-4}$   
 Determine the value of  $\epsilon_x$ ,  $\epsilon_y$  and  $\gamma_{xy}$  at the point  $(x, y) = (1.2, 1.6)$ .
- b) Consider the following state of stress and strain:  
 $\sigma_{xx} = x^2$ ,  $\sigma_{yy} = y^2$ ,  $\epsilon_{xy} = -3xy$ ,  $\sigma_{zz} = \epsilon_{xz} = \epsilon_{yz} = 0$   
 Determine whether the compatibility equations are satisfied. [6+10]
- 2.a) Calculate the element load vector due to uniform temperature change  $\Delta T = 110^\circ \text{C}$ , for a one dimensional linear element with the following data:  
 $E = 180 \text{ Gpa}$ ,  $a = 300 \text{ mm}^2$  and  $\alpha = 8 \times 10^{-6}$  per  $^\circ\text{C}$ .
- b) Derive the shape function for a 1-D quadratic element in terms of global and natural coordinates. [6+10]
3. For the beam shown in fig. 1, determine the beam deflection and member end loads. [16]

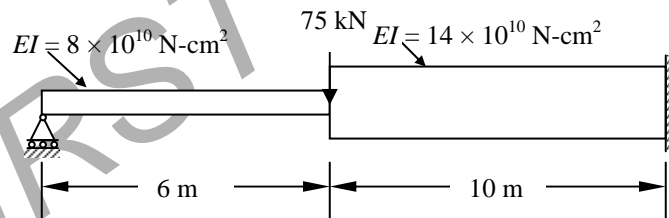


Fig. 1

- 4.a) Calculate the jacobian matrix for a triangular element with nodal coordinates as  $(1, 1)$ ,  $(6, 4)$  and  $(3, 5)$ .
- b) Calculate the element stresses for the element shown in fig. 2 for plane stress condition when nodal displacements are as given below:  
 $q_1 = 0$ ,  $q_2 = 0$ ,  $q_3 = 0.001 \text{ mm}$ ,  $q_4 = 0.002 \text{ mm}$ ,  $q_5 = -0.003 \text{ mm}$  and  $q_6 = 0.002 \text{ mm}$   
 $E = 200 \text{ GPa}$ ,  $\nu = 0.25$ , thickness = 20mm. [6+10]

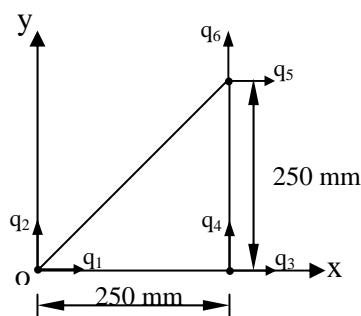


Fig.2

- 5.a) Derive the material property matrix for a axisymmetric element.  
 b) Formulate element equations for the axisymmetric element shown in fig.3. [6+10]

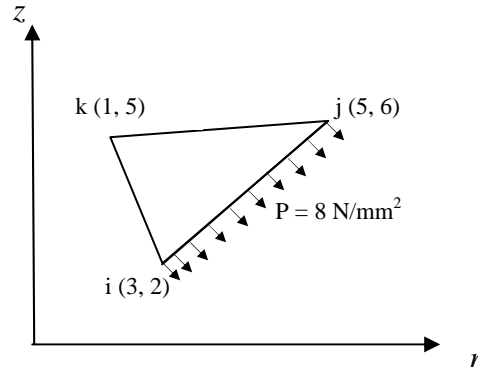
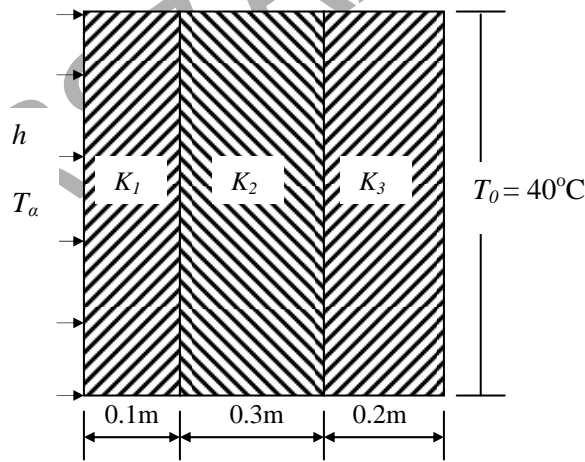


Fig. 3

- 6.a) Evaluate the integral  $\int_{-1}^{+1} (6\xi^3 + 3\xi^2) d\xi$  using two point Gaussian quadrature methods.  
 b) In a quadrilateral element, the nodal coordinates in cm are given below:  
 $x_1 = 6; x_2 = 2; x_3 = 3; x_4 = 9$   
 $y_1 = 8; y_2 = 5; y_3 = 2; y_4 = 5$   
 Determine the global coordinates corresponding to  $\xi = -1.0 \eta = 0.75$  on the parent element. [6+10]
7. For the composite wall shown in fig. 4. Determine the temperature distribution. [16]



- $K_1 = 25 \text{ W/m } ^\circ\text{C}$
- $K_2 = 80 \text{ W/m } ^\circ\text{C}$
- $K_3 = 25 \text{ W/m } ^\circ\text{C}$
- $h = 300 \text{ W/m}^2 \text{ } ^\circ\text{C}$
- $T_a = 1500 \text{ } ^\circ\text{C}$

Fig. 4

- 8.a) State the method used for obtaining natural frequencies and corresponding eigen vectors.  
 b) Determine the natural frequencies of a cantilever beam shown in shown in Fig. 5. using 2 beam elements with consistent mass matrix. [6+10]

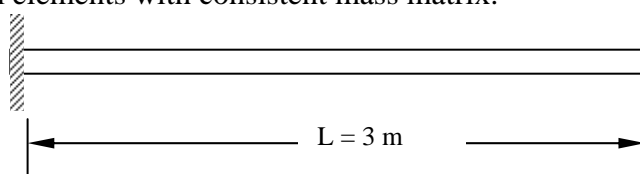


Fig. 5

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- 1.a) Explain the various steps involved in solving a problem using finite element method.  
 b) Determine whether the following state of strain is physically realizable:

$$\varepsilon_{xx} = c(x^2 + y^2), \quad \varepsilon_{yy} = cy^2, \quad \varepsilon_{xy} = 3cxy, \quad \varepsilon_{zz} = \varepsilon_{xz} = \varepsilon_{yz} = 0. \quad [6+10]$$

2. For the two stepped bar shown in fig. 1, which is at room temperature is fixed at one end and a rigid wall at the other end. The temperature is then raised by  $50^\circ\text{C}$ . Determine the displacements at nodes 2 and 3, stresses in the two sections. [16]

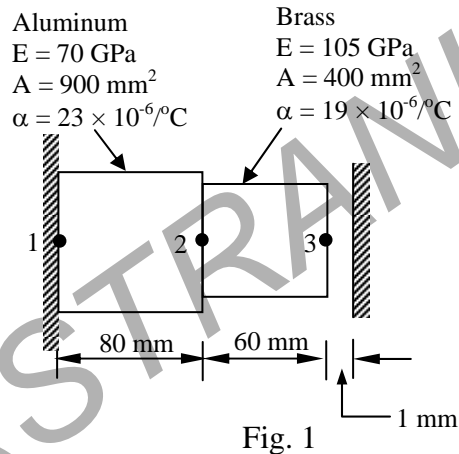


Fig. 1

- 3.a) Distinguish between essential and natural boundary conditions in beam problems.  
 b) Find the nodal displacements and slope at the mid-point of element (2) of the shaft shown in fig.2. The shaft is assumed to be supported in bearings regarded as fixed support at A and B.  $E = 210\text{ GPa}$ ;  $I = 4 \times 10^6\text{ N-m}^2$ . [6+10]

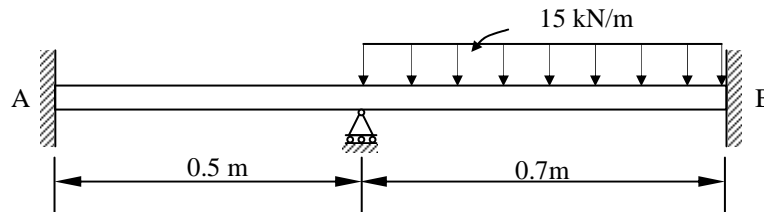


Fig. 2

4. For the two-dimensional loaded plate shown in fig. 3 determine the displacements of nodes 1 and 2 and the element stresses using plane stress condition. [16]

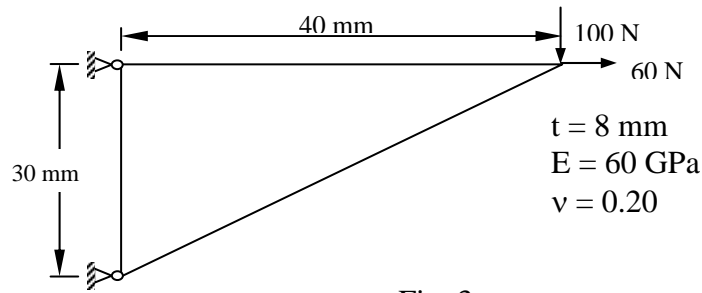


Fig. 3

5. A long cylinder with outside diameter 140mm is fitted on to a shaft of outside diameter of 100 mm. The cylinder is raised to a temperature of  $60^\circ\text{C}$ . Using two elements of 20 mm length, formulate the element matrices for the cylinder taking  $E = 210 \text{ GPa}$ ;  $\nu = 0.25$  and  $\alpha = 5 \times 10^{-6} \text{ per } ^\circ\text{C}$ . Also find nodal displacements. [16]
6. For the quadratic, isoparametric triangular element shown in fig. 4 map the point  $\xi = 0.4$  and  $\eta = 0.3$  on the parent element to the corresponding point on the distorted element. [16]

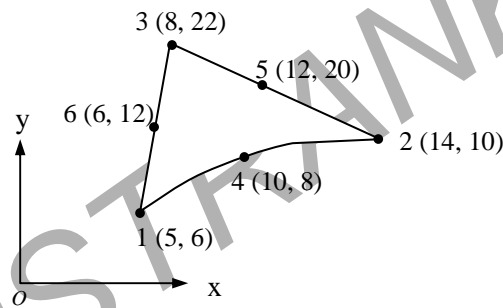


Fig. 4

- 7.a) What are essential and natural boundary conditions in steady state heat transfer problems.
- b) Formulate the finite element equations for triangular torsion element shown in fig.5 [6+10]

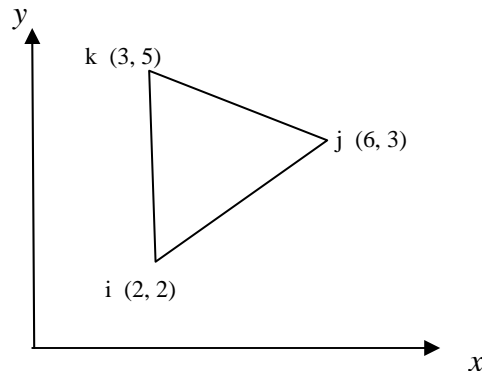


Fig. 5



- 8.a) Derive the consistent mass matrix and lumped mass matrices for a one dimensional bar element.
- b) Consider the eigen value problem where

$$[K] = \begin{bmatrix} 6 & -1 & 4 \\ -3 & 8 & -2 \\ 4 & -2 & 6 \end{bmatrix}; \quad [M] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Compute the eigen values and eigen vectors.

[8+8]

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